Energy Conversion – Exam I

School of Engineering – Dep. of Industrial & Mechanical Eng.

Name: Solut° Key
Date: Friday, March 28th 2014; 06:00 PM
Location: ENG Auditorium
Instructor: Dr. Wassim Habchi
Notes: No documents allowed
Time: 2 hours

[___ / 100]

Problem I (20 points)

An Air-conditioning system is used to maintain a house at a constant temperature of 21°C in a summer day where the outside temperature is 37°C. The house is estimated to be gaining heat through the walls, roof and windows at a rate of 48MJ/h. If the COP of the Air-conditioning system is 2.5, determine:

a) The power consumption of the compressor **in kW** **(5 points)**

b) The reversible power that would be consumed by the compressor **in kW** **(5 points)**

c) The rate of Irreversibility **in kW** **(5 points)**

d) The second-law efficiency of this Air-conditioning system **(5 points)**

Solution:

$T_L = 21°C = 294\,K \qquad T_H = 37°C = 310\,K \qquad \dot{Q}_L = 48\,MJ/h \qquad COP = 2.5$

a) $COP = \frac{\dot{Q}_L}{\dot{W}_{comp}} \Rightarrow \dot{W}_{comp} = \frac{\dot{Q}_L}{COP} = \frac{48}{2.5} = 19.2\,MJ/h = \frac{19.2\times10^3}{3600}\,kW = \boxed{5.33\,kW}$

b) $\dot{W}_{rev} = \frac{\dot{Q}_L}{COP_{rev}} = \frac{\dot{Q}_L}{\frac{1}{\frac{T_H}{T_L}-1}} = \frac{48000/3600}{\frac{1}{\frac{310}{294}-1}} = \boxed{0.7256\,kW}$

c) $\dot{I} = \dot{W} - \dot{W}_{rev} = 5.33 - 0.7256 = \boxed{4.6044\,kW}$

d) $\eta_{II} = \frac{\dot{W}_{rev}}{\dot{W}_{actual}} = \frac{0.7256}{5.33} = 0.136 = \boxed{13.6\,\%}$

or $\eta_{II} = \frac{COP}{COP_{rev}} = \frac{2.5}{\frac{1}{\frac{310}{294}-1}} = 0.136 = \boxed{13.6\,\%}$

Problem II (25 points)

At the beginning of the compression process of an air-standard Diesel cycle operating with a compression ratio of 18, the temperature is 300K and the pressure is 0.1MPa. The cutoff ratio for the cycle is 2 and the ideal-gas constant for Air is R = 0.287 Kj/Kg.K.

a) Sketch a P-v diagram of the corresponding cycle (**5 points**)

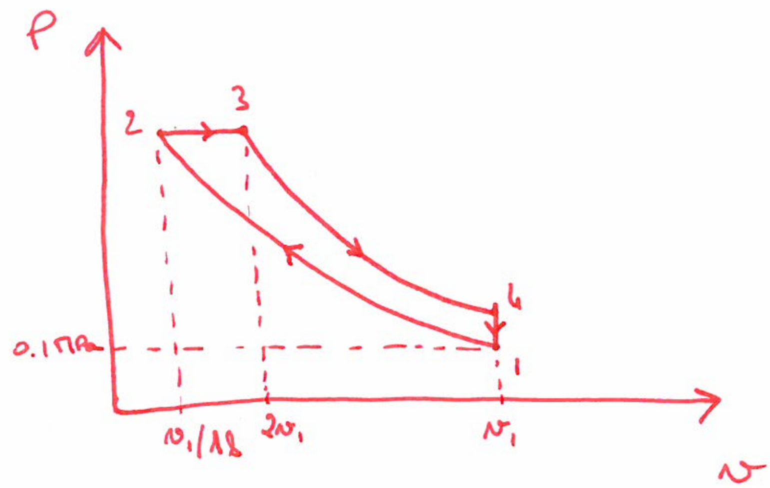
b) Determine the temperature and pressure at the end of each process (**10 points**)

c) Determine the thermal efficiency (**5 points**)

d) Determine the mean effective pressure in ~~M~~KPa (**5 points**)

Solution:

a)



b) $\frac{v_{r_2}}{v_{r_1}} = \frac{v_2}{v_1} = \frac{1}{18} \Rightarrow v_{r_2} = \frac{v_{r_1}}{18}$ but @ 300K, $v_{r_1} = 621.2$

$\Rightarrow v_{r_2} = \frac{621.2}{18} = 34.51 \Rightarrow$ $\boxed{T_2 = 888.26\text{ K}}$

And $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow P_2 = P_1 \frac{V_1}{V_2} \times \frac{T_2}{T_1} = 0.1 \times 18 \times \frac{888.26}{300} = \boxed{5.33\text{MPa}}$

$\boxed{P_3 = P_2 = 5.33\text{MPa}}$ & $V_3 = 2V_2$

since $\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2} \Rightarrow T_3 = \frac{\cancel{P_3} V_3}{\cancel{P_2} V_2} \times T_2 = 2 \times 888.26 = \boxed{1776.52\text{K}}$

finally: $\frac{v_{r_3}}{v_{r_4}} = \frac{v_3}{v_4} = \frac{v_3}{v_2} \times \frac{v_2}{v_4} = \frac{2}{18} = \frac{1}{9}$ but $v_{r_3} = 4.017$ @ T_3

$\rightarrow v_{r_4} = 9 \times 4.017 = 36.153 \rightarrow$ $\boxed{T_4 = 883.97\text{K}}$

and $\frac{P_4}{T_4} = \frac{P_1}{T_1}$ since $V_4 = V_1 \Rightarrow P_4 = P_1 \frac{T_4}{T_1} = 0.1 \times \frac{883.97}{300} = \boxed{0.295\text{MPa}}$

c) $\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$

with $q_{out} = u_4 - u_1 = 661.25 - 214.07 = 447.18$ KJ/Kg

$q_{in} = h_3 - h_2 = 1999 - 930.98 = 1068.02$ KJ/Kg.

From A-17: $u_1 = 214.07$ KJ/Kg

$u_4 = 661.25$ KJ/Kg

$h_2 = 930.98$ KJ/Kg

$h_3 = 1999$ KJ/Kg

$\rightarrow \eta_{th} = 1 - \frac{661.25 - 214.07}{1999 - 930.98} = 0.5813 = \boxed{58.13\ \%}$

d) $MEP = \frac{W_{net,out}}{V_1 - V_2} = \frac{w_{net,out}}{v_1 - v_2} = \frac{q_{net,in}}{v_1 - v_2} = \frac{q_{in} - q_{out}}{v_1 - v_2} = \frac{q_{in} - q_{out}}{v_1\left(1 - \frac{1}{18}\right)}$

$MEP = \frac{q_{in} - q_{out}}{v_1 \times 17/18}$

but $v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{0.1 \times 10^3} = 0.861\ m^3/Kg$

$\Rightarrow MEP = \frac{1068.02 - 447.18}{0.861 \times 17/18} = \boxed{763.48\ KPa}$

Problem III (20 points)

At the beginning of the compression process of an air-standard dual cycle with a compression ratio of 18, the temperature is 300 K and the pressure is 0.1 MPa. The pressure ratio for the constant volume part of the heating process is 1.5. The volume ratio for the constant pressure part of the heating process is 1.2. Assuming constant specific heats for the Air with c_p=1.005kJ/kg.K and R=0.287kJ/kg.K and $K = 1.4$

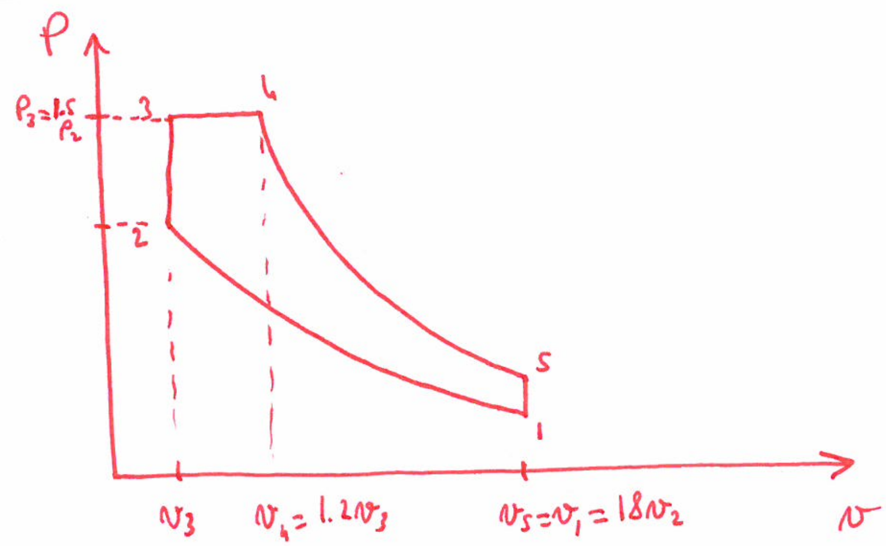
a) Sketch a P-v diagram of the corresponding cycle (**5 points**)

b) Determine the thermal efficiency (**10 points**)

c) Determine the mean effective pressure **in MPa** (**5 points**)

Solution:

a)



b) $\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$ but $q_{out} = u_5 - u_1 = c_v (T_5 - T_1)$

$q_{in} = q_{2-3} + q_{3-4} = c_v (T_3 - T_2) + c_p (T_4 - T_3)$

$\Rightarrow \eta_{th} = 1 - \frac{c_v (T_5 - T_1)}{c_v (T_3 - T_2) + c_p (T_4 - T_3)}$ but $c_v = -R + c_p = -0.287 + 1.005$

$\boxed{c_v = 0.718 \text{ kJ/kg.K}}$

but $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{K-1} = 18^{0.4} \Rightarrow T_2 = 300 \times 18^{0.4} = \boxed{953.3 \text{ K}}$

and $\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{K} = 18^{1.4} \Rightarrow P_2 = 0.1 \times 18^{1.4} = \boxed{5.72 \text{ MPa}}$

but $P_3 = 1.5 P_2 = 1.5 \times 5.72 = 8.58 \text{ MPa}$

& $\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \Rightarrow T_3 = \frac{P_3 \cancel{v_3}}{P_2 \cancel{v_2}} \times T_2 = 1.5 \times 953.3 = \boxed{1429.95 \text{ K}}$

and $\frac{\cancel{P_4} v_4}{T_4} = \frac{\cancel{P_3} v_3}{T_3} \Rightarrow T_4 = \frac{v_4}{v_3} T_3 = 1.2 T_3 = 1.2 \times 1429.95 = \boxed{1715.94 \text{ K}}$

Finally: $\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{k-1} = \left(\frac{v_4}{v_3} \times \frac{v_3}{v_5}\right)^{k-1} = \left(1.2 \times \frac{1}{18}\right)^{0.4}$

$\Rightarrow T_5 = T_4 \times \left(\frac{1.2}{18}\right)^{0.4} = 1715.96 \times \left(\frac{1.2}{18}\right)^{0.4} = \boxed{580.85\ K}$

$\Rightarrow \eta_{th} = 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)} = 1 - \frac{0.718(580.85 - 300)}{0.718(1429.95 - 993.3) + 1.005(1715.96 - 1429.95)}$

$\Rightarrow \eta_{th} = 1 - \frac{201.65}{629.65} = 0.6797 = \boxed{67.97\%}$

c) $MEP = \frac{w_{net,out}}{v_1 - v_2} = \frac{q_{in} - q_{out}}{v_1\left(1 - \frac{1}{18}\right)}$

but $v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{0.1 \times 10^3} = 0.861\ m^3/kg$

$\Rightarrow MEP = \frac{629.65 - 201.65}{0.861 \times 17/18} = 526.337\ kPa \simeq \boxed{0.526\ MPa}$

Problem IV (35 points)

Air enters the compressor of a gas power plant at 100 kPa, 300 K. The air is compressed in two stages to 900 kPa, with intercooling to 300 K between the stages at a pressure of 300 kPa. The turbine inlet temperature is 1480 K and the expansion occurs in two stages, with reheat to 1420 K between the stages at a pressure of 300 kPa. The compressor and turbine stage efficiencies are 84 and 82%, respectively (for both stages). The net power developed is 1.8 MW.

a) Sketch a T-s diagram of the corresponding cycle (**5 points**)

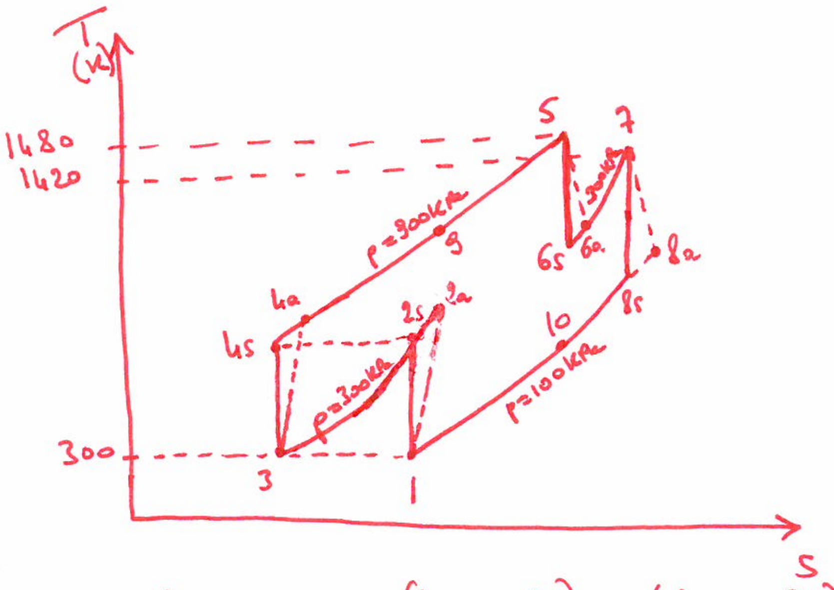
b) Determine the volumetric flow rate, in m³/s, at the inlet of each ~~compressor~~ turbine stage (**5 points**)

c) Determine the thermal efficiency of the cycle and the back work ratio (**10 points**) if $R = 0.287$ kJ/kg.K

d) If a regenerator of effectiveness 80% is added to the cycle, determine the new back work ratio and thermal efficiency of the cycle (**10 points**)

Solution:

a)



b) $\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$ with $q_{out} = (h_{8a} - h_1) + (h_{2a} - h_3)$

$q_{in} = (h_5 - h_{4a}) + (h_7 - h_{6a})$

but $\frac{P_{r2s}}{P_{r1}} = \frac{P_2}{P_1} = \frac{300}{100} = 3 \rightarrow P_{r2s} = 3P_{r1} = 3 \times 1.386 = \boxed{4.158}$

$\rightarrow h_{2s} = 411.26$ kJ/kg but $\eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1} = 0.84$

$\rightarrow h_{2a} = \frac{h_{2s} - h_1}{0.84} + h_1 = \frac{411.26 - 300.19}{0.84} + 300.19 = \boxed{432.42 \text{ kJ/kg}}$

since $\frac{P_4}{P_3} = \frac{P_2}{P_1}$ and $T_3 = T_1$ and $\eta_{c2} = \eta_{c1}$ $\rightarrow$ $\boxed{h_3 = h_1 = 300.19 \text{ kJ/kg}, \; h_{4a} = h_{2a} = 432.42 \text{ kJ/kg}}$

@⑤: $T_5 = 1480$ K $\longrightarrow$ $\boxed{h_5 = 1611.79 \text{ KJ/Kg}}$

$P_{r5} = 568.8$

but $\frac{P_{r5}}{P_{r6}} = \frac{P_5}{P_{6s}} = \frac{300}{300} = 3 \Rightarrow P_{r6} = \frac{P_{r5}}{3} = \frac{568.8}{3} = \boxed{189.6}$

$\Rightarrow \boxed{h_{6s} = 1201.49 \text{ KJ/Kg}}$

but $\eta_{T_1} = 0.82 = \frac{h_5 - h_{6a}}{h_5 - h_{6s}} \Rightarrow h_{6a} = h_5 - 0.82(h_5 - h_{6s})$

$= 1611.79 - 0.82(1611.79 - 1201.49)$

$\boxed{h_{6a} = 1275.34 \text{ KJ/Kg.}}$

@⑦: $T_7 = 1420$ K $\longrightarrow$ $\boxed{h_7 = 1539.44 \text{ KJ/Kg}}$ and $P_{r7} = 478$

but $\frac{P_{r7}}{P_{r8}} = \frac{P_7}{P_{8s}} = \frac{300}{100} = 3 \rightarrow P_{r8} = \frac{P_{r7}}{3} = \frac{478}{3} = 159.33$

$\Rightarrow \boxed{h_{8s} = 1145.93 \text{ KJ/Kg}}$

and $h_{8a} = h_7 - 0.82(h_7 - h_{8s}) = 1539.44 - 0.82(1539.44 - 1145.93)$

$\boxed{h_{8a} = 1216.76 \text{ KJ/Kg}}$

$\Rightarrow \eta_{th} = 1 - \frac{(1216.76 - 300.19) + (432.42 - 300.19)}{(1611.79 - 432.42) + (1539.44 - 1275.34)} = 1 - \frac{1048.8}{1443.47}$

$\boxed{\eta_{th} = 0.2734 = 27.34\%}$

And $r_{bw} = \frac{W_{comp}}{W_{turb}} = \frac{(h_{4a} - h_3) + (h_{2a} - h_1)}{(h_5 - h_{6a}) + (h_7 - h_{8a})} = \frac{(432.42 - 300.19) + (432.42 - 300.19)}{(1611.79 - 1275.34) + (1539.44 - 1216.76)}$

$r_{bw} = \frac{264.46}{658.13} = 0.4012 = \boxed{40.12\%}$

c) $\dot{W}_{net,out} = \dot{m}\, w_{net,out} = \dot{m}\,(w_{turb} - w_{pump})$

$$\Rightarrow \dot{m} = \frac{\dot{W}_{net,out}}{w_{turb} - w_{pump}} = \frac{1.8 \times 10^3}{653.13 - 264.46} = 4.56\ kg/s$$

but $\dot{m} = \dot{m}_5 = \dot{m}_7 = \frac{\dot{V}_5}{v_5} = \frac{\dot{V}_7}{v_7}$

$\Rightarrow \dot{V}_5 = \dot{m}\, v_5$ and $\dot{V}_7 = \dot{m}\, v_7$

$\dot{V}_5 = \dot{m}\,\frac{RT_5}{P_5}$ $\dot{V}_7 = \dot{m}\,\frac{RT_7}{P_7}$

$\dot{V}_5 = 4.56 \times 0.287 \times \frac{1480}{900}$ & $\dot{V}_7 = 4.56 \times 0.287 \times \frac{1420}{300}$

$\boxed{\dot{V}_5 = 2.15\ m^3/s}$ and $\boxed{\dot{V}_7 = 6.19\ m^3/s}$

d) the back-work ratio is unchanged : $\boxed{r_{bw} = 40.12\%}$

However $q_{in} = h_5 - h_8 + h_7 - h_{6a}$

with $\varepsilon = \frac{h_8 - h_{4a}}{h_{8a} - h_{4a}} \Rightarrow h_8 = \varepsilon(h_{8a} - h_{4a}) + h_{4a}$

$= 0.8(1216.76 - 432.42) + 432.42$

$= \boxed{1059.892\ kJ/kg}$

$\Rightarrow q_{in} = 1611.79 - 1059.89 + 1539.44 - 1275.34 = 816\ kJ/kg$

$$\Rightarrow \eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{w_{turb} - w_{comp}}{q_{in}} = \frac{653.13 - 264.46}{816} = 0.484 = \boxed{48.4\%}$$

TABLE A–17

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	s° kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559
210	209.97	0.3987	149.69	1512.0	1.34444
220	219.97	0.4690	156.82	1346.0	1.39105
230	230.02	0.5477	164.00	1205.0	1.43557
240	240.02	0.6355	171.13	1084.0	1.47824
250	250.05	0.7329	178.28	979.0	1.51917
260	260.09	0.8405	185.45	887.8	1.55848
270	270.11	0.9590	192.60	808.0	1.59634
280	280.13	1.0889	199.75	738.0	1.63279
285	285.14	1.1584	203.33	706.1	1.65055
290	290.16	1.2311	206.91	676.1	1.66802
295	295.17	1.3068	210.49	647.9	1.68515
298	298.18	1.3543	212.64	631.9	1.69528
300	300.19	1.3860	214.07	621.2	1.70203
305	305.22	1.4686	217.67	596.0	1.71865
310	310.24	1.5546	221.25	572.3	1.73498
315	315.27	1.6442	224.85	549.8	1.75106
320	320.29	1.7375	228.42	528.6	1.76690
325	325.31	1.8345	232.02	508.4	1.78249
330	330.34	1.9352	235.61	489.4	1.79783
340	340.42	2.149	242.82	454.1	1.82790
350	350.49	2.379	250.02	422.2	1.85708
360	360.58	2.626	257.24	393.4	1.88543
370	370.67	2.892	264.46	367.2	1.91313
380	380.77	3.176	271.69	343.4	1.94001
390	390.88	3.481	278.93	321.5	1.96633
400	400.98	3.806	286.16	301.6	1.99194
410	411.12	4.153	293.43	283.3	2.01699
420	421.26	4.522	300.69	266.6	2.04142
430	431.43	4.915	307.99	251.1	2.06533
440	441.61	5.332	315.30	236.8	2.08870
450	451.80	5.775	322.62	223.6	2.11161
460	462.02	6.245	329.97	211.4	2.13407
470	472.24	6.742	337.32	200.1	2.15604
480	482.49	7.268	344.70	189.5	2.17760
490	492.74	7.824	352.08	179.7	2.19876
500	503.02	8.411	359.49	170.6	2.21952
510	513.32	9.031	366.92	162.1	2.23993
520	523.63	9.684	374.36	154.1	2.25997
530	533.98	10.37	381.84	146.7	2.27967
540	544.35	11.10	389.34	139.7	2.29906
550	555.74	11.86	396.86	133.1	2.31809
560	565.17	12.66	404.42	127.0	2.33685
570	575.59	13.50	411.97	121.2	2.35531
580	586.04	14.38	419.55	115.7	2.37348
590	596.52	15.31	427.15	110.6	2.39140
600	607.02	16.28	434.78	105.8	2.40902
610	617.53	17.30	442.42	101.2	2.42644
620	628.07	18.36	450.09	96.92	2.44356
630	638.63	19.84	457.78	92.84	2.46048
640	649.22	20.64	465.50	88.99	2.47716
650	659.84	21.86	473.25	85.34	2.49364
660	670.47	23.13	481.01	81.89	2.50985
670	681.14	24.46	488.81	78.61	2.52589
680	691.82	25.85	496.62	75.50	2.54175
690	702.52	27.29	504.45	72.56	2.55731
700	713.27	28.80	512.33	69.76	2.57277
710	724.04	30.38	520.23	67.07	2.58810
720	734.82	32.02	528.14	64.53	2.60319
730	745.62	33.72	536.07	62.13	2.61803
740	756.44	35.50	544.02	59.82	2.63280
750	767.29	37.35	551.99	57.63	2.64737
760	778.18	39.27	560.01	55.54	2.66176
780	800.03	43.35	576.12	51.64	2.69013
800	821.95	47.75	592.30	48.08	2.71787
820	843.98	52.59	608.59	44.84	2.74504
840	866.08	57.60	624.95	41.85	2.77170
860	888.27	63.09	641.40	39.12	2.79783
880	910.56	68.98	657.95	36.61	2.82344
900	932.93	75.29	674.58	34.31	2.84856
920	955.38	82.05	691.28	32.18	2.87324
940	977.92	89.28	708.08	30.22	2.89748
960	1000.55	97.00	725.02	28.40	2.92128
980	1023.25	105.2	741.98	26.73	2.94468
1000	1046.04	114.0	758.94	25.17	2.96770
1020	1068.89	123.4	776.10	23.72	2.99034
1040	1091.85	133.3	793.36	23.29	3.01260
1060	1114.86	143.9	810.62	21.14	3.03449
1080	1137.89	155.2	827.88	19.98	3.05608
1100	1161.07	167.1	845.33	18.896	3.07732
1120	1184.28	179.7	862.79	17.886	3.09825
1140	1207.57	193.1	880.35	16.946	3.11883
1160	1230.92	207.2	897.91	16.064	3.13916
1180	1254.34	222.2	915.57	15.241	3.15916
1200	1277.79	238.0	933.33	14.470	3.17888
1220	1301.31	254.7	951.09	13.747	3.19834
1240	1324.93	272.3	968.95	13.069	3.21751

TABLE A–17

Ideal-gas properties of air (*Concluded*)

T K	h kJ/kg	P_r	u kJ/kg	v_r	s° kJ/kg · K	T K	h kJ/kg	P_r	u kJ/kg	v_r	s° kJ/kg · K
1260	1348.55	290.8	986.90	12.435	3.23638	1600	1757.57	791.2	1298.30	5.804	3.52364
1280	1372.24	310.4	1004.76	11.835	3.25510	1620	1782.00	834.1	1316.96	5.574	3.53879
1300	1395.97	330.9	1022.82	11.275	3.27345	1640	1806.46	878.9	1335.72	5.355	3.55381
1320	1419.76	352.5	1040.88	10.747	3.29160	1660	1830.96	925.6	1354.48	5.147	3.56867
1340	1443.60	375.3	1058.94	10.247	3.30959	1680	1855.50	974.2	1373.24	4.949	3.58335
1360	1467.49	399.1	1077.10	9.780	3.32724	1700	1880.1	1025	1392.7	4.761	3.5979
1380	1491.44	424.2	1095.26	9.337	3.34474	1750	1941.6	1161	1439.8	4.328	3.6336
1400	1515.42	450.5	1113.52	8.919	3.36200	1800	2003.3	1310	1487.2	3.994	3.6684
1420	1539.44	478.0	1131.77	8.526	3.37901	1850	2065.3	1475	1534.9	3.601	3.7023
1440	1563.51	506.9	1150.13	8.153	3.39586	1900	2127.4	1655	1582.6	3.295	3.7354
1460	1587.63	537.1	1168.49	7.801	3.41247	1950	2189.7	1852	1630.6	3.022	3.7677
1480	1611.79	568.8	1186.95	7.468	3.42892	2000	2252.1	2068	1678.7	2.776	3.7994
1500	1635.97	601.9	1205.41	7.152	3.44516	2050	2314.6	2303	1726.8	2.555	3.8303
1520	1660.23	636.5	1223.87	6.854	3.46120	2100	2377.7	2559	1775.3	2.356	3.8605
1540	1684.51	672.8	1242.43	6.569	3.47712	2150	2440.3	2837	1823.8	2.175	3.8901
1560	1708.82	710.5	1260.99	6.301	3.49276	2200	2503.2	3138	1872.4	2.012	3.9191
1580	1733.17	750.0	1279.65	6.046	3.50829	2250	2566.4	3464	1921.3	1.864	3.9474

Note: The properties P_r (relative pressure) and v_r (relative specific volume) are dimensionless quantities used in the analysis of isentropic processes, and should not be confused with the properties pressure and specific volume.

Source: Kenneth Wark, *Thermodynamics*, 4th ed. (New York: McGraw-Hill, 1983), pp. 785–86, table A–5. Originally published in J. H. Keenan and J. Kaye, *Gas Tables* (New York: John Wiley & Sons, 1948).